



Overview

- What is cryptography?
- Classic cryptosystems
 - The Caesar cipher
 - Monoalphabetic replacement cipher
 - The one-time pad
- Types of cryptosystems
 - Codes vs. ciphers
 - Symetric-key vs. assymmetric-key
 - Hybrid cryptosystems



What is Cryptography?

- Text defines **Cryptography** as the science of designing and analyzing cryptosystems which are used to disguise messages so that only certain people can see through the disguise
- A classic cryptosystem: the Caesar cipher
 - Replace every ‘A’ in the message with a ‘D’
 - Replace every ‘B’ in the message with a ‘E’
 - Replace every ‘C’ in the message with a ‘F’
 - Etc.



The Caesar Cipher

- Camouflage the message “ATTACK AT DAWN” by writing “DWWDFN DW GDZQ”
- “ATTACK AT DAWN” is **plaintext**
- “DWWDFN DW GDZQ” is **ciphertext**
- **Encryption** is the process used to convert plaintext into ciphertext
- **Decryption** is the process used to convert ciphertext into plaintext



The Key to a Cryptosystem

- Assumptions:
 - Encryption and decryption algorithms are public
 - Their results depend on some value known as a **key**
 - Protection is based solely on the secrecy of the key
 - Encryption for the Caesar cipher = “shift forward by n ”
 - Decryption for the Caesar cipher = “shift backwards by n ”
 - The key for the Caesar cipher is n
 - Encryption: $C_i = (P_i + n) \bmod 26$
 - Decryption: $P_i = (C_i - n) \bmod 26$



The Keyspace for a Cryptosystem

- For the Caesar cipher, any value from the set $\{1, 2, \dots, 25\}$ can be a key
- The set of usable keys is referred to as a cryptosystem's **keyspace**
- Cryptosystems with a small keyspace are vulnerable to a **brute-force search** for the proper key



What is Cryptanalysis?

- **Cryptanalysis** the science of attacking cryptosystems
 - Deduce the key and/or
 - recover plaintext
- Assume adversary knows the ciphertext and encryption algorithm



Cryptanalysis of the Caesar Cipher

- Ciphertext = “GRR MGAR OY JOBOJKJ OT ZNXKK VGXZY”
- Perform decryption with each possible key:
 - Plaintext (if key is 1): FQQ LFZQ NX INANIJI NS YMWJJ UFWYX
 - Plaintext (if key is 2): EPP KEYP MW HMZMHIH MR XLVII TEVXW
 - Plaintext (if key is 3): DOO JDXO LV GLYLGHG LQ WKUHH SDUWV
 - Plaintext (if key is 4): CNN ICWN KU FKXKFGF KP VJTGG
RCTVU
 - Plaintext (if key is 5): BMM HBVM JT EJWJEFE JO UISFF QBSUT
 - Plaintext (if key is 6): ALL GAUL IS DIVIDED IN THREE PARTS
 - Plaintext (if key is 7): ZKK FZTK HR CHUHCDC HM SGQDD
OZQSR
 - ...
 - Plaintext (if key is 26): GRR MGAR OY JOBOJKJ OT ZNXKK
VGXZY

- Only one of the plaintexts above (the one corresponding to a key of 6) makes sense



The Monoalphabetic Replacement Cipher

- Similar to the Caesar cipher but with much larger key space
- A key is any permutation of the 26 letters of the alphabet

– Example:

JQPLMZKOWHANXIEURYTGSDVFCB

- Defines a **cipher alphabet**:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
J	Q	P	L	M	Z	K	O	W	H	A	N	X	I	E	U	R	Y	T	G	S	F	D	V	C	B



The Monoalphabetic Replacement Cipher - Encryption

- Plaintext (*by Thomas Jefferson*):
 - “I prefer freedom with danger to slavery with ease.”
- Cipher alphabet

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
J	Q	P	L	M	Z	K	O	W	H	A	N	X	I	E	U	R	Y	T	G	S	F	D	V	C	B

- Encryption: replace each plaintext letter with the corresponding cipher letter from the cipher alphabet
- Examples:
 - Replace every “A” in the plaintext with a “J”
 - Replace every “B” in the plaintext with a “Q”
 - Replace every “C” in the plaintext with a “P”
 - Etc.



The Monoalphabetic Replacement Cipher – Encryption (cont)

- Plaintext:
 - “I prefer freedom with danger to slavery with ease.”
- Cipher alphabet:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
J	Q	P	L	M	Z	K	O	W	H	A	N	X	I	E	U	R	Y	T	G	S	F	D	V	C	B

- Ciphertext:
 - “W uymzmy zymmlex dwgo ljikmy ge tnjfmvc dwgo mjtm.”



The Monoalphabetic Replacement Cipher - Decryption

- Ciphertext:
 - “W uymzmy zymmlex dwgo ljikmy ge tnjfmvc dwgo mjt看.”
- Cipher alphabet

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
J	Q	P	L	M	Z	K	O	W	H	A	N	X	I	E	U	R	Y	T	G	S	F	D	V	C	B

- Decryption: replace each plaintext letter with the corresponding cipher letter from the cipher alphabet
- Plaintext:
 - “I prefer freedom with danger to slavery with ease.”



The Monoalphabetic Replacement Cipher - Keyspace

- Key = any permutation of the 26 letters of the alphabet
- Keyspace contains $26!$ elements
 - 403,291,461,126,605,635,584,000,000
- Exhaustive search at one trillion keys per second takes:
 - 400 trillion seconds
 - More than 12 million years
- It is fairly easy to perform cryptanalysis on this cipher, but not using exhaustive search



The Monoalphabetic Replacement Cipher – Weak Keys

- Some keys result in better-disguised ciphertext than others:
 - Using JQPLMZKOWHANXIEURYTGSDVCB as a key gives:
“W uymzmy zymmlex dwgo ljikmy ge tnjfmvc dwgo mjt看.”
 - Using ABCDEFGHIJKLMNOPQRSTUVWXYZ as a key gives:
“I prefer freedom with danger to slavery with ease.”
 - Using ABCDEFGHIJKLMNOPQRSTUVWXYZ as a key gives:
“I prefer freedom with danger to slaverz with ease.”
- Keys that produce weak ciphertext are called **weak** keys
- Weak keys need not be a problem so long as:
 - They are not used
 - The vast majority of the keys are not weak



One-Time Pads

- Unbreakable
- Sender and receiver must generate a large, non-repeating set of truly random key letters
 - E.g. IPKLPSFHGQYPWKQMSVCX...
- Sender uses each key letter on the pad to encrypt one letter of plaintext
 - $C_i = (P_i + K_i) \bmod 26$
- Receiver uses each key letter on the pad to decrypt one letter of ciphertext
 - $P_i = (C_i - K_i) \bmod 26$



One-Time Pad Encryption - Example

- One time pad:
IPKLPSFHGQYPWKQMSVCX...

- Plaintext:
“ATTACKATDAWN”

- Ciphertext:
“JJEMSDGBKRVD”

$A (1) + I (9) \pmod{26} = J (10)$	$A (1) + F (6) \pmod{26} = G (7)$
$T (20) + P (16) \pmod{26} = J (10)$	$T (20) + H (8) \pmod{26} = B (2)$
$T (20) + K (11) \pmod{26} = E (5)$	$D (4) + G (7) \pmod{26} = K (11)$
$A (1) + L (12) \pmod{26} = M (13)$	$A (1) + Q (17) \pmod{26} = R (18)$
$C (3) + P (16) \pmod{26} = S (19)$	$W (23) + Y (25) \pmod{26} = V (22)$
$K (11) + S (19) \pmod{26} = D (4)$	$N (14) + P (16) \pmod{26} = D (4)$



One-Time Pad Decryption - Example

- One time pad:
IPKLPSFHGQYPWKQMSVCX...
- Ciphertext:
“JJEMSDGBKRVD”
- Plaintext:
“ATTACKATDAWN”
 $J (10) - I (9) \bmod 26 = A (1)$
 $J (10) - P (16) \bmod 26 = T (20)$
 $E (5) - K (11) \bmod 26 = T (20)$
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One-Time Pad - Security

- Why is it an unbreakable encryption algorithm?
 - Assume the adversary doesn't know any of the key letters on the one-time pad
 - If they were generated truly randomly then all key letters are equally likely in each position
 - So when the adversary sees the ciphertext, “JJEMSDGBKRVD”
 - All plaintexts are equally possible:
 - JJEMSDGBKRVD = ATTACKATDAWN for IPKLPSFHGQYP
 - JJEMSDGBKRVD = ELVISISALIVE for EXIDZUNAYIZY
 -
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One-Time Pad - Security (cont)

- Every plaintext message is equally possible
- No way for an adversary to determine which plaintext is correct
- A truly random key sequence added to a nonrandom plaintext produces a truly random ciphertext
- No algorithm will enable the adversary to choose the proper plaintext with better than random probability



One-Time Pads - Drawbacks

- Key must be as long as the message
- Security depends on adversary never obtaining a copy of the pad
 - Pad must be distributed securely to sender and receiver
 - Pad must be destroyed immediately after use to lessen the likelihood that old messages will be compromised
- Security depends on using the cryptosystem properly
 - Pad must be generated truly randomly (pseudo-random won't due)
 - No part of the pad can ever be reused



Types of Cryptosystems

- Codes, ciphers, or a combination of the two
- Ciphers (e.g. the Caesar cipher)
 - Transform each plaintext block into a ciphertext block
 - **Block** is a fixed-size unit on which a cryptosystem operates
 - Single character (e.g. Caesar cipher)
 - Two or more characters



Ciphers

- **Substitution** ciphers apply some function to the plaintext block and key to produce a block of ciphertext which replaces the plaintext (e.g. the Caesar cipher)
- **Transposition** ciphers shuffle the blocks into a new order that depends on the plaintext block and key

A	T	T	A	C
K		A	T	
D	A	W	N	

= “AKDT ATAWATNC”

A	K	D
T		A
T	A	W
A	T	N
C		

= “ATTACK AT DAWN”



Codes

- Sender and receiver each have a copy of a **codebook** which specifies one or more **codewords** for each word that might be used in a message:

Word	Codeword
AT	September
ATTACK	March
ATTACK	December
DAWN	April
DAWN	October
(null)	July
(null)	January



Codes – Encryption and Decryption

- Plaintext:
 - “ATTACK AT DAWN”
- Ciphertext:
 - “March September October” or
 - “March September April” or
 - “July December January September April July” or ...
- Codewords can be random numbers, strings of characters, or other symbols



Types of Cryptosystems (cont)

- Symmetric-key
 - Same key used for encryption and decryption
 - Typically used for bulk encryption
- Asymmetric-key (or public-key)
 - Different key used for encryption and decryption
 - Usually not used for bulk encryption
- Hybrid cryptosystems



Symmetric-key Cryptosystems

- Standard use of a symmetric-key cryptosystem:
 - Sender and receiver agree on a secret key
 - Must be done securely!
 - Messages are encrypted by the sender with the shared key and decrypted by the receiver with the shared key
 - Note: Users need to have a previously-established shared secret to communicate securely



Public-Key Cryptosystems

- Standard use of a public-key cryptosystem:
 - Generate a public-key/private-key pair
 - Disseminate your public key widely
 - Keep your private key secret
 - Anybody can encrypt a message to you using your public key
 - Only you can decrypt the message using your private key
 - Note: unlike symmetric-key cryptosystems, users don't need to have a previously-established shared secret to communicate securely



Public-Key Cryptosystems (cont)

- Standard use of a public-key cryptosystem:
 - Digital signatures - proof of authorship of a document or agreement with its contents
 - User encrypts a document with his private key to create a digital signature
 - Anybody can verify the digital signature by using the signer's public key
 - Only the signer can produce his signature, and he can't reasonably claim he didn't sign a document bearing his signature
 - Note: unlike symmetric-key cryptosystems, users can create authentic, unforgeable, nonreusable, nonrepudiable digital signatures



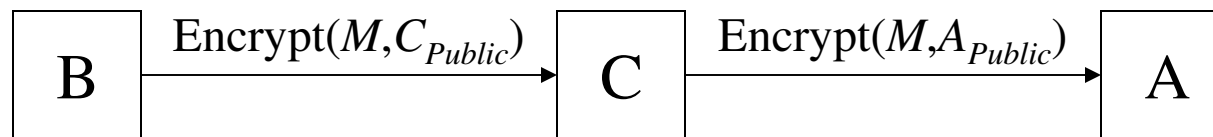
Public-Key Cryptosystems (cont)

- In order for a public-key cryptosystem to work:
 - For every message, M , $\text{Decrypt}(\text{Encrypt}(M, A_{Public}), A_{Private}) = M$
 - For every pair of users, A and B , $(A_{Public}, A_{Private})$ and $(B_{Public}, B_{Private})$ must be distinct
 - Deriving $A_{private}$ from A_{Public} or the plaintext from the ciphertext is difficult
 - Key generation, encryption, and decryption routines must be relatively fast



Public-Key Cryptosystems - Problems

- Problem #1 - Man in the Middle:
 - Recall - everybody should know A 's public key
 - So if B wants to send a message, M , to A then B needs to encrypt M with A_{Public}
 - What if an adversary, C , is able to trick B into thinking that C_{Public} is A_{Public} ?



- A and B think their messages are secure, but C can read them
- Public-key cryptography depends heavily on knowing to whom a public key belongs



Public-Key Cryptosystems - Problems

- Problem #2 - Known Ciphertext:
 - Recall - everybody should know A 's public key
 - So if C sees an encrypted message, $\text{Encrypt}(M, A_{\text{Public}})$ from B to A
 - C can choose a message, M'
 - Encrypt M' with A 's public key to get $\text{Encrypt}(M', A_{\text{Public}})$
 - Compare $\text{Encrypt}(M', A_{\text{Public}})$ with $\text{Encrypt}(M, A_{\text{Public}})$, if they match then C knows the message B sent to A
 - This is a serious problem if the number of possible plaintext messages is small enough to allow exhaustive search



Hybrid Cryptosystems

- Symmetric-key cryptosystems:
 - Good for bulk data, but require shared secrets
- Public-key cryptosystems:
 - Don't require any shared secrets, but too slow for bulk encryption
- Hybrid cryptosystems:
 - Given a message M
 - Choose a key, K , at random to be used with a symmetric-key algorithm
 - Encrypt K with the recipient's public key
 - Encrypt M with K
 - Send to recipient:





Hybrid Cryptosystems (cont)

- Hybrid cryptosystems:

$\text{Encrypt}(K, A_{Public})$	$\text{Encrypt}(M, K)$
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- Recipient decrypts first part of the message with his/her private key to learn K
- Recipient uses K to decrypt the remainder of the message
- Result: Doesn't require any shared secrets, and good for bulk encryption



Summary

- **Cryptography** is the science of designing and analyzing cryptosystems which are used to disguise messages so that only certain people can see through the disguise
- **Cryptanalysis** is the science of attacking cryptosystems
- Classic cryptosystems include the **Caesar cipher**, **monoalphabetic replacement cipher**, and **one-time pad**
- **Symmetric-key** cryptosystems are useful for bulk data encryption but required a shared secret
- **Public-key** cryptosystems are much slower but don't require shared secrets and support digital signatures
- **Hybrid** cryptosystems are good for bulk encryption and don't require any shared secrets